Electron Correlation in a Two-Sublattice Hubbard Model*

Dipan K. Ghosh
Tata Institute of Fundamental Research, Bombay, India
(Received 5 August 1970)

We discuss the two-sublattice Hubbard model using a new Green's-function technique proposed by Sawada. It is found that the initial two bands split into four because of the interaction. The perturbed density of states is seen to be obtainable from the unperturbed case by a simple transformation, the "shape-factor" modulating the shape of the bands being different from unity only in the magnetic state. It is seen that in the atomic limit the results agree with the exact calculation, and in the limit of weak coupling one gets back the earlier known results.

I. INTRODUCTION

In recent years, Hubbard's narrow-energy-band model[1] in the theory of ferromagnetism in transition metals has received considerable attention. In the first of his series of papers on the subject, he treated the finite bandwidth case by using a one-particle Green's-function technique.[2] The Hamiltonian retains only that part of the interaction which takes into account the correlation between electrons on the same site, and is written

$$\mathcal{H}=\sum_{i,j}\sum_{\sigma} T_{ij}C_{i\sigma}^{\dagger}C_{j\sigma}+I\sum_{i} n_{i\sigma}n_{i-\sigma}-\lambda\sum_{i,\sigma} n_{i\sigma}, \qquad (1)$$

where, in terms of Bloch energies,

$$T_{ij}=(1/N)\sum_{\vec{k}} \epsilon_{k} e^{i\vec{k}\cdot(\vec{R}_i-\vec{R}_j)},$$

and the index i denotes a Wannier function on the site i, and $C_i^{\dagger}$ is the creation operator. A term with chemical potential λ has been added to conserve the number of particles. In spite of the inherent simplicity of the one-particle Green's-function formalism, the procedure of decoupling the higher Green's functions is somewhat arbitrary. Kemeney[3] used two-particle Green's functions instead, but his formalism too has the same uncertainty in regard to the decoupling. Recently, Sawada[4] has proposed an alternative Green's-function method which goes some way in clarifying the meaning of the truncation procedures and in partly removing the aforementioned ambiguity. This has been applied with considerable success by Oguchi [5] in his treatment of the Anderson model.

In this paper we study, using this technique, a two-sublattice Hubbard model. Langer, Plischke, and Mattis[6] have solved this model within an extended Hartree-Fock type of approximation in the two-particle Green's function, and have predicted two phase transitions, one from a paramagnetic to a magnetic state, and the other from a metal to an insulator state. Earlier, Johansson and Berggren[7] had considered Overhauser's spin-density-wave state of a general pitch for a linear chain with a half-filled band, and had obtained some of the results of Langer *et al.* in one dimension.

In Sec. II we review Sawada's new Green's-function approach and consider its connection with the conventional double-time Green's functions. In Sec. III we explicitly write down the equation of motion for the Green's function for the two-sublattice Hubbard Hamiltonian, and discuss the various approximations made. In Sec. IV, we discuss the nature of the solutions.

II. MATHEMATICAL TECHNIQUE

As we have pointed out above, the conventional double-time Green's function uses a rather arbitrary decoupling procedure. In some cases the nature of the solution varies, in some qualitative aspects, with different decoupling procedures. The Sawada scheme does not need any such decoupling, and the meaning of the approximations is somewhat clearer.

Consider a many-fermion system, described by a Hamiltonian

$$\mathcal{H}=\sum_{\vec{k}\sigma}\epsilon_{k}n_{\vec{k}\sigma}+\tfrac{1}{4}\sum_{i,j,l,m} V(i,j,l,m)C_i^{\dagger}C_j^{\dagger}C_lC_m. \qquad (2)$$

The operators $C_i(t)$, in the Heisenberg representation, are given by

$$\begin{aligned} C_i(t) &= e^{i\mathcal{H}t}C_i(0)e^{-i\mathcal{H}t} \\ &= C_i(0)+it[\mathcal{H}, C_i]_{t=0} \\ &\quad +[(it)^2/2!][\mathcal{H},[\mathcal{H}, C_i]]_{t=0}+\cdots . \end{aligned} \qquad (3)$$

Equation (3) suggests that the operator $C_i(t)$ is composed of operators $C_j(0)$, $C_j^{\dagger}(0)C_k(0)C_l(0),\ldots$, etc., because of the presence of the interaction term V in the Hamiltonian (2). We expand the operator at a given time t in terms of operators which arise from the various terms of (3). We write

$$\begin{aligned} \theta(t)C_i(t) &= \sum_{j} G_{ij}(t)C_j(0) \\ &\quad +\sum_{j,l,m} K_{i,jlm}(t)\circ C_j^{\dagger}(0)C_l(0)C_m(0)\circ+\cdots , \end{aligned} \qquad (4)$$

where $\theta(t)$ is the step function. The notation $\circ\cdots\circ$ is defined as

$$\circ AB\cdots VW\circ = AB\cdots VW - \langle AB\cdots VW\rangle - A\langle B\cdots VW\rangle + B\langle A\cdots VW\rangle - AB\langle\cdots VW\rangle - \cdots - AB\cdots V\langle W\rangle. \tag{5}$$

The sign on each term of the right-hand side of (3) is minus if the number of permutations of the Fermi operators is even, as in Wick's theorem.

From (4) one obtains,

$$\langle\{\theta(t)C_i(t);\, C_n^\dagger(0)\}\rangle = \sum_j G_{ij}(t)\langle\{C_j(0);\, C_n^\dagger(0)\}\rangle \langle\{\circ C_j^\dagger(0)C_l(0)C_m(0)\circ\ ;\ C_n^\dagger(0)\}\rangle + \cdots, + \sum_{jlm} K_{i,jlm}(t) \tag{6}$$

where the curly bracket is an anticommutator. The second and the higher terms on the right are identically zero by the definition of $\circ\cdots\circ$, so that we get

$$G_{in} = \langle\{\theta(t)C_i(t);\, C_n^\dagger(0)\}\rangle. \tag{7}$$

Similarly we can show that

$$K_{i,jlm} = \langle\{[\,\{\theta(t)C_i(t);\, C_m^\dagger(0)\},\, C_l(0)];\, C_j^\dagger(0)\}\rangle. \tag{8}$$

One observes that the coefficient G_{in} is the same as the one-particle Green's function of Zubarev, so that a solution for G_{in} will give us interesting quantities like the density of states, etc. In particular, the correlation function $\langle C_n^\dagger(0)C_i(0)\rangle$ will be given by

$$\langle C_n^\dagger(0)C_i(0)\rangle = \int \frac{G_{in}(\omega+i\epsilon) - G_{in}(\omega-i\epsilon)}{e^{\beta(\omega-\lambda)}+1}\,d\omega \quad (\epsilon\to 0^+), \tag{9}$$

where $G_{in}(\omega)$ is the time Fourier transform of $G_{in}(t)$. The equation of motion which follows from (4) is

$$i\frac{d}{dt}\theta(t)C_i(t) = i\delta(t)C_i(0) + \sum_j G_{ij}(t)[C_j, \mathcal{H}] + \sum_{jlm} K_{i,jlm}(t)[\circ C_j^\dagger C_l C_m \circ, \mathcal{H}] + \cdots. \tag{10}$$

The coefficient $K_{i,jlm}$ is related to the two-particle Green's function, and similarly the higher ones are related to the multiparticle Green's functions. In order to solve Eq. (10), we shall use the approximation of truncating the infinite sum on the right-hand side at some suitable point. Using the Hartree-Fock approximation will correspond to terminating the series at the very first term, while the second term contains two-body correlations.

III. TWO-SUBLATTICE MODEL

We consider a simple cubic lattice and decompose it *ab initio* into two sublattices called A and B. This is done with a view to obtaining antiferromagnetic ordering. The Hamiltonian for such a two-sublattice Hubbard system can be written as

$$\mathcal{H} = -\sum_{\vec{i}\in A}\sum_{\vec{j}\in B} T_{ij}\{C_{\vec{i}\uparrow}^\dagger C_{\vec{j}\uparrow} + C_{\vec{j}\downarrow}^\dagger C_{\vec{i}\downarrow} + C_{\vec{i}\downarrow}^\dagger C_{\vec{j}\downarrow} + C_{\vec{j}\uparrow}^\dagger C_{\vec{i}\uparrow}\} + I\sum_{\vec{i}} n_{\vec{i}\uparrow}n_{\vec{i}\downarrow} - \lambda\sum_i (n_{\vec{i}\uparrow} + n_{\vec{i}\downarrow}), \tag{11}$$

where the sums on $\vec{i}$ and $\vec{j}$ are over nearest neighbors in a simple cubic structure. For this Hamiltonian, one finds

$$[C_{\vec{k}\sigma}^A, \mathcal{H}] = -\sum_{\vec{j}\in B} T_{kj}C_{\vec{j}\sigma}^B + IC_{\vec{k}\sigma}^A n_{\vec{k}-\vec{\sigma}}^A - \lambda C_{\vec{k}\sigma}^A,$$
$$[n_{\vec{k}\sigma}^A, \mathcal{H}] = -\sum_{\vec{j}\in B} T_{\vec{k}j}(C_{\vec{k}\sigma}^{\dagger A}C_{\vec{j}\sigma}^B - C_{\vec{j}\sigma}^{\dagger B}C_{\vec{k}\sigma}^A). \tag{12}$$

Thus, following the methods of Sec. II, we will consider the Heisenberg operator $C_{\vec{k}\sigma}^A(t)$ to be composed of the operators $C_{\vec{k}\sigma}^A(0)$, $C_{\vec{k}\sigma}^B(0)$, $C_{\vec{k}\sigma}^A(0)\ n_{\vec{k}-\vec{\sigma}}^A(0)$ and $C_{\vec{k}\sigma}^B(0)\ n_{\vec{k}\sigma}^B(0)$ of the Schrödinger representation. It must be noted that of the three fermion operators, we have retained only those which correspond to the same site index. This naturally means that the multiparticle correlations are not adequately taken into account. However, this is consistent with the fact that in the Hubbard Hamiltonian, only the integrals $(ii\,|\,1/r\,|\,ii)\equiv I$, corresponding to the interaction between electrons on the same site, are considered important. Hence, before an attempt to include multiparticle correlations is made, one has to investigate the effect of integrals like $(ij\,|\,1/r\,|\,ij)$.[8,9] Thus, we write

$$\theta(t)C_{\vec{i}\vec{\sigma}}^A(t) = \sum_{\vec{j}\in A}\sum_{\vec{\sigma}'} G_{ij}^{\vec{\sigma}\vec{\sigma}'}C_{\vec{j}\vec{\sigma}'}^A + \sum_{\vec{j}\in A}\sum_{\vec{\sigma}'} F_{i,j}^{\vec{\sigma}\vec{\sigma}}\circ C_{\vec{j}\vec{\sigma}'}^A n_{\vec{j}-\vec{\sigma}}^A\circ + \sum_{\vec{j}\in B}\sum_{\vec{\sigma}'} K_{ij}^{\vec{\sigma}\vec{\sigma}'}C_{\vec{j}\sigma'}^B + \sum_{\vec{j}\in B}\sum_{\vec{\sigma}'} L_{ij}^{\vec{\sigma}\vec{\sigma}'}\circ C_{\vec{j}\sigma}^B n_{\vec{j}-\vec{\sigma}'}^B\circ. \tag{13}$$

The equation of motion for the Green's functions G, F, K, and L follow from (7), (8), (10), (12), and (13):

$$\left(i\frac{d}{dt} + \lambda - In_-^A\right)G_{\vec{i}\vec{p}}^{\vec{\sigma}\vec{\sigma}'} = i\delta(t)\delta_{\vec{\sigma}\vec{\sigma}'}\delta_{\vec{i}\vec{p}} + In_-^A(1-n_-^A)F_{\vec{i}\vec{p}}^{\vec{\sigma}\vec{\sigma}'} + \sum_{\vec{k}\in B} T_{kp}K_{\vec{i}\vec{k}}^{\vec{\sigma}\vec{\sigma}'}, \tag{14}$$

$$\left(i\frac{d}{dt} + \lambda - I(1-n_+^A)\right)F_{\vec{i}\vec{p}}^{\vec{\sigma}\vec{\sigma}'} = IG_{\vec{i}\vec{p}}^{\vec{\sigma}\vec{\sigma}'}, \tag{15}$$

$$\left(i\frac{d}{dt} + \lambda - In_-^B\right)K_{\vec{i}\vec{p}}^{\vec{\sigma}\vec{\sigma}'} = In_-^B(1-n_-^B)L_{\vec{i}\vec{p}}^{\vec{\sigma}\vec{\sigma}'} + \sum_{\vec{j}\in A} G_{\vec{i}\vec{j}}^{\vec{\sigma}\vec{\sigma}'}T_{jp} \tag{16}$$

$$\left(i\frac{d}{dt} + \lambda - I(1-n_+^B)\right)L_{\vec{i}\vec{p}}^{\vec{\sigma}\vec{\sigma}'} = IK_{\vec{i}\vec{p}}^{\vec{\sigma}\vec{\sigma}'}, \tag{17}$$

where

$$n_\pm^A = \langle n_{\pm\vec{\sigma}}^A\rangle \quad\text{and}\quad n_\pm^B = \langle n_{\pm\vec{\sigma}}^B\rangle. \tag{18}$$

In writing down Eqs. (14)–(17), we have used the

fact that the Hamiltonian (11) conserves both spin and particle number, so that ensemble averages of type $\langle C^{\dagger}_{\vec{\sigma}} C_{-\vec{\sigma}}\rangle$ can be put equal to zero. Further, we have assumed the correlation functions $\langle C^{\dagger A}_{\vec{i}-\vec{\sigma}}\ C^{B}_{\vec{j}-\sigma}\rangle$ to be real. This assumption is not really necessary, but has been introduced for simplicity. Let[6]

$$\alpha=\langle n^{A}_{\vec{i}\vec{\sigma}}\rangle=\langle n^{B}_{\vec{i}-\vec{\sigma}}\rangle\,,\quad \gamma=\langle n^{A}_{\vec{i}-\vec{\sigma}}\rangle=\langle n^{B}_{\vec{i}\vec{\sigma}}\rangle. \tag{19}$$

This allows for a possible antiferromagnetic ordering on sublattices A and B. We assume translational invariance within each sublattice. The set of equations (14)–(17) can be solved by introducing fourier transforms

$$G_{\vec{i}_1\vec{i}_2}(t)=\frac{1}{2\pi N}\sum_{\kappa_1\kappa_2}\int d\omega\, e^{-i\omega t+i\vec{\kappa}_1\cdot\vec{i}_1-i\vec{\kappa}_2\cdot\vec{i}_2}G(\kappa_1,\kappa_2;\omega),$$
$$T_{ij}=\frac{1}{N}\sum_{\vec{\kappa}} e^{i\vec{\kappa}\cdot(\vec{i}-\vec{j})}T(\vec{\kappa}). \tag{20}$$

The solution for $G(\vec{\kappa}_1,\vec{\kappa}_2;\omega)$ can be written as

$$G(\vec{\kappa}_1,\vec{\kappa}_2;\omega)=\frac{i}{2\pi}\frac{\delta(\vec{\kappa}_1\vec{\kappa}_2)}{F_1-T^2(\vec{\kappa})/F_2}, \tag{21}$$

where

$$F_1=(\omega+\lambda-I\gamma)-\frac{I^2\gamma(1-\gamma)}{\omega+\lambda-I(1-\alpha)}\,,$$
$$F_2=(\omega+\lambda-I\alpha)-\frac{I^2\alpha(1-\alpha)}{\omega+\lambda-I(1-\gamma)}\,. \tag{22}$$

In Sec. IV we shall examine the nature of the solutions (21) and (22) for the case of a half-filled band, viz., $n^A+n^B=1$.

IV. NATURE OF SOLUTION FOR HALF-FILLED BAND CASE

A. Atomic Limit

In the limit of zero bandwidth, $T(k)=0$ (the origin of energy having been chosen such that $T_{ii}=0$). In this case, it is possible to obtain an exact solution for the conventional Green's functions.[1] From (21) it can be seen that

$$G(\vec{\kappa}_1,\vec{\kappa}_2,\omega)=\frac{i}{2\pi}\,\delta(\vec{\kappa}_1\vec{\kappa}_2)\frac{1}{2}\Big(\frac{1}{\omega+\lambda-I\gamma+I(\alpha\gamma)^{1/2}}$$
$$+\frac{1}{\omega+\lambda-I\gamma-I(\alpha\gamma)^{1/2}}\Big), \tag{23}$$

where we have used the condition

$$\alpha+\gamma=1. \tag{24}$$

The correlation functions $\langle C^{\dagger A}_{\vec{\kappa}}; C^{A}_{\vec{\kappa}}\rangle$ can be obtained from the Green's functions (23) and Eq. (9). One obtains

$$\langle C^{\dagger A}_{\vec{\kappa}}, C^{A}_{\vec{\kappa}}\rangle=\tfrac{1}{2}\{f[I\gamma+I(\alpha\gamma)^{1/2}-\lambda]$$
$$+f[I\gamma-I(\alpha\gamma)^{1/2}-\lambda]\} \tag{25}$$

with $f(x)=(1+e^{\beta x})^{-1}$. The correlation $\langle C^{\dagger B}_{\vec{\kappa}\vec{\sigma}}\ C^{B}_{\vec{\kappa}\vec{\sigma}}\rangle$ is obtained by the interchange $\alpha\rightleftharpoons\gamma$. Thus our self-consistency conditions are

$$1=\tfrac{1}{2}\{f[I\gamma+I(\alpha\gamma)^{1/2}-\lambda]+f[I\gamma-I(\alpha\gamma)^{1/2}-\lambda]$$
$$+f[I\alpha+I(\alpha\gamma)^{1/2}-\lambda]+f[I\alpha-I(\alpha\gamma)^{1/2}-\lambda]\}, \tag{26a}$$
$$x=\tfrac{1}{2}\{f[I\gamma+I(\alpha\gamma)^{1/2}-\lambda]+f[I\gamma-I(\alpha\gamma)^{1/2}-\lambda]$$
$$-f[I\alpha+I(\alpha\gamma)^{1/2}-\lambda]-f[I\alpha-I(\alpha\gamma)^{1/2}-\lambda]\}, \tag{26b}$$

where $x=\alpha-\gamma$. Equations (26a) and (26b) give the solution $x=0$ and $\lambda=\frac{1}{2}I$. This latter condition has been proved rigorously for the Hubbard model.[10] From (23), we see that the system behaves as though it has two energy levels $\frac{1}{2}I$ and $-\frac{1}{2}I$ containing half an electron each. This is precisely the solution one obtains on solving the problem exactly.[11] Thus, we see that the new scheme gives us the proper atomic limit. This is only to be expected, because in this limit all multiparticle correlations are zero.

B. Weak-Coupling Limit

Yet another interesting limiting case of the solutions (21) and (22) is obtained when the interaction strength I is very small. In this case we can write $G(\vec{\kappa}_1,\vec{\kappa}_2;\omega)$ as

$$G(\vec{\kappa}_1,\vec{\kappa}_2;\omega)=\frac{i}{2\pi}\frac{\delta(\vec{\kappa}_1\vec{\kappa}_2)}{(\omega+\lambda-I\gamma)-T^2(\vec{\kappa})/(\omega+\lambda-I\alpha)}\,. \tag{27}$$

Expressed differently,

$$G(\vec{\kappa}_1,\vec{\kappa}_2;\omega)=\frac{i}{2\pi}\frac{\delta(\vec{\kappa}_1\vec{\kappa}_2)}{E(\vec{\kappa})}(\omega+\lambda-I\alpha)$$
$$\times\Big(\frac{1}{\omega+\lambda-Iy/2-E(\vec{\kappa})}$$
$$-\frac{1}{\omega+\lambda-Iy/2+E(\vec{\kappa})}\Big), \tag{28}$$

where

$$E(\vec{\kappa})=[\tfrac{1}{4}I^2x^2+T^2(\vec{\kappa})]^{1/2},\quad x=\alpha-\gamma,\quad y=\alpha+\gamma. \tag{29}$$

One observes that the Eqs. (28) and (29) are the same as the results obtained by Langer *et al.*[6] using a conventional Green's function with a Hartree-Fock–type decoupling. The significance of this result lies in the fact that Langer *et al.* obtained a fairly good agreement of the ground-state energy of a one-dimensional Hubbard system with the exact results of Lieb and Wu,[12] at all values of coupling constant, "becoming almost exact agreement in the weak coupling limit." This calculation shows that the treatment of Langer *et al.* is expected to be particularly good in this limit.

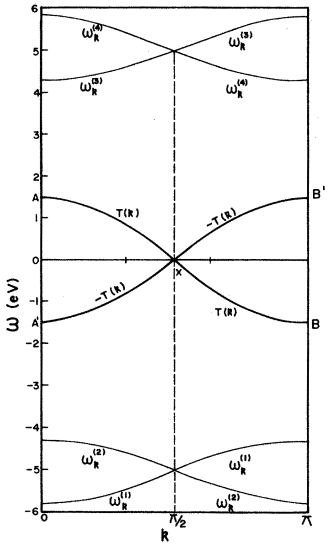

FIG. 1. Sketch of the pseudoparticle band structure $\omega_{k\sigma}^{(1)}$, $\omega_{k\sigma}^{(2)}$, $\omega_{k\sigma}^{(3)}$, and $\omega_{k\sigma}^{(4)}$. The curves AXB and $A'XB'$ represent the unperturbed band structure. The parameters corresponding to this figure are $I=10$ eV, $T_0=\frac{1}{2}$, and $x=0$.

C. Finite-Bandwidth Case

In what follows, we shall use the result $\lambda=\frac{1}{2}I$ for the case of general bandwidth. With this, the solutions (21) and (22) can be written as

$$G_{\sigma\sigma}^{AA}(\vec{k}_1,\vec{k}_2;\omega)=\frac{i}{2\pi}\,\delta(\vec{k}_1\vec{k}_2)\sum_i\frac{A_{k\sigma}^{(i)}}{\omega-\omega_{k\sigma}^{(i)}}\,. \tag{30}$$

Here

$$\omega_{k\sigma}^{(1)}>\omega_{k\sigma}^{(2)}>\omega_{k\sigma}^{(3)}>\omega_{k\sigma}^{(4)}$$

are the roots of the equation

$$\omega^4-[\tfrac{1}{2}I^2+T^2(\vec{k})]\omega^2$$

$$+[\tfrac{1}{4}I^4x^4-\tfrac{1}{4}I^4x^2+\tfrac{1}{16}I^4+\tfrac{1}{4}I^2x^2T^2(\vec{k})]=0, \tag{31}$$

and $A_{k\sigma}^{(1)}$, $A_{k\sigma}^{(2)}$, etc., are given by

$$A_{k\sigma}^{(i)}=\frac{\partial}{\partial\omega}\,\frac{F_1F_2-T^2(\vec{k})}{F_2}\bigg|_{\omega=\omega_{k\sigma}^{(i)}},\quad i=1,2,3,4. \tag{32}$$

In the absence of any interaction, $I=0$, the system has two energy levels with the dispersion law $\omega\sim T(\vec{k})$ and $\omega\sim -T(\vec{k})$. This is also seen from a more elementary consideration by putting $I=0$ in the Hamiltonian (11) and solving for the Green's function G^{AA} exactly. What happens in this case is that, due to the doubling of the translational vector caused by the decomposition of the lattice into two sublattices, the portion of the dispersion curve between the new Brillouin-zone boundary and the original Brillouin zone, gives the new branch $\omega\sim T(\vec{k})$.

In the perturbed case ($I\neq 0$), the density of states per atom of type A of spin $\vec{\sigma}$ is given by

$$\rho_{\vec{\sigma}}(\vec{\omega})=\frac{1}{N}\sum_{i=1}^{4}\sum_{\vec{k}}A_{k\vec{\sigma}}^{(i)}\,\delta(\omega-\omega_{k\vec{\sigma}}^{(i)}). \tag{33}$$

It is observed that the strength factors $A_{k\vec{\sigma}}^{(i)}$ obey the sum rule $\sum_i A_{k\vec{\sigma}}^{(i)}=1$. If one had $A_{k\vec{\sigma}}^{(i)}=1$ for $i=1$, 4, Eq. (33) would be appropriate to a band structure having four bands with the dispersion law $\omega\sim\omega_{k\vec{\sigma}}^{(i)}$. The effect of the factors $A_{k\vec{\sigma}}^{(i)}$ cannot be given a simple interpretation beyond saying that they reduce the density of states in each band in such a way that the total number of states per atom is just one and not four, as would have been the case if all $A_{k\vec{\sigma}}^{(i)}$ were unity. In Fig. 1, we have plotted the general form of the bands $\omega_{k\vec{\sigma}}^{(i)}$; ($i=1,2,3,4$), in the nonmagnetic case. For simplicity we have chosen [111] direction. For a simple cubic lattice, $T(\vec{k})=T_0(\cos\kappa_x+\cos\kappa_y+\cos\kappa_z)$, and we have taken $T_0=\frac{1}{2}$, corresponding to a bandwidth of three.

Noting that

$$\delta(g(x))=\sum_n\frac{\delta(x-a_n)}{g'(a_n)}, \tag{34}$$

we can write

$$\rho_{\vec{\sigma}}(\omega)=\frac{1}{N}\sum_{\vec{k}}\delta[g(x)],$$

where

$$g(x)=\frac{\omega^4-[\frac{1}{2}I^2+T^2(\vec{k})]\omega^2+[\frac{1}{4}I^4x^4-\frac{1}{2}I^4x^2+\frac{1}{16}I^4+\frac{1}{4}I^2x^2T^2(\vec{k})]}{\omega^3-\frac{1}{2}Ix\omega^2-\frac{1}{2}I^2\omega+[\frac{1}{4}I^3x^3-\frac{1}{8}I^3x]}\equiv\frac{f_1(x)}{f_2(x)}\,. \tag{35}$$

From (35) it follows that

$$\rho_{\vec{\sigma}}(\omega)=\frac{|f_2(x)|}{|(\omega^2-\frac{1}{4}I^2x^2)^{1/2}[\omega^4-\frac{1}{2}I^2\omega^2+\frac{1}{4}I^4x^4-\frac{1}{4}I^2x^2+\frac{1}{16}I^4]^{1/2}|}\,P\left(\frac{\omega^4-\frac{1}{2}I^2\omega^2+\frac{1}{4}I^4x^4-\frac{1}{4}I^2x^2+\frac{1}{16}I^4}{\omega^2-\frac{1}{4}I^2x^2}\right), \tag{36}$$

where

$$P(\omega^2)=\frac{1}{N}\sum_{\vec{k}}\delta[\omega^2-T^2(\vec{k})] \tag{37}$$

is the unperturbed density of states. Thus, we see that the perturbed density of states is obtained from the unperturbed density of states by the transformations (36) and (37). The "shape factor" multiplying $P[f(\omega)]$ in (33) becomes unity when $x=0$, i.e., in the nonmagnetic state. If there is a residual magnetic moment the perturbed density of states consists of four bands separated by gaps, the shape of the bands having been altered owing to the interaction.

ACKNOWLEDGMENTS

It is a pleasure to thank Dr. C. K. Majumdar and Professor Stig Lundquist for many helpful comments and suggestions. The author is indebted to Dr. B. Johansson for a critical reading of the manuscript and for many useful discussions in connection with Dr. Johansson's work cited in Ref. 7. Interesting discussions with Dr. A. Lopez and Dr. I. Hulthen are gratefully acknowledged. Finally, the author wishes to thank IAEA and UNESCO for hospitality and support at the International Centre for Theoretical Physics, Trieste during the period when most of this work was completed.

---

*Work supported in part by the Swedish International Development Authority (SIDA).

[1]J. Hubbard, Proc. Roy. Soc. (London) A276, 238 (1963).

[2]D. N. Zubarev, Usp. Fiz. Nauk 71, 71 (1960) [Soviet Phys. Usp. 3, 320 (1960)].

[3]G. Kemeney, Ann. Phys. (N. Y.) 32, 69 (1965).

[4]K. Sawada (unpublished); see Ref. 5.

[5]A. Oguchi, Progr. Theoret. Phys. (Kyoto) 43, 257 (1970).

[6]W. Langer, M. Plischke, and D. Mattis, Phys. Rev. Letters 23, 1448 (1969); and Phys. Rev. Letters 24, 635(E) (1970).

[7]B. Johansson and K. F. Berggren, Phys. Rev. 181, 855 (1969).

[8]R. Kishore and S. K. Joshi, Phys. Rev. 186, 484 (1969).

[9]D. K. Ghosh, Phys. Letters 32A, 429 (1970).

[10]D. Mattis and L. Landovitz (unpublished); see Ref. 6.

[11]While comparing with the solution of Hubbard (Ref. 1) in the atomic limit, one must shift the zero of energy in his case to $T_0+\frac{1}{2}I=0$.

[12]E. Lieb and F. Wu, Phys. Rev. Letters 20, 1445 (1968).